Limits Review

1) $\lim_{x \to c} \frac{x^4 - x^3 + 1}{x^2 + 9} = \frac{c^4 - c^3 + 1}{c^2 + 9}$

2) $\lim_{x \to -4} (x+3)^{1998} = 1$

3) $\lim_{x \to 1} (x^3 + 3x^2 - 2x - 17) = -15$

4) $\lim_{x \to \frac{1}{2}} (\text{int } x) = 0$

5) $\lim_{x \to -2} \left(\frac{1}{x+2}\right)$ DNE

6) $\lim_{x \to 0} \frac{(4+x)^2 - 16}{x}$

$\lim_{x \to 0} (4+x)^2 - 16 = 0 \quad \lim_{x \to 0} x = 0$

$\lim_{x \to 0} \frac{16 + 8x + x^2 - 16}{x}$

$\lim_{x \to 0} (8 + x) = 8$

7) $\lim_{x \to 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2}$

$\lim_{x \to 0} (5x^3 + 8x^2) = 0 \quad \lim_{x \to 0} (3x^4 - 16x^2) = 0$

$\lim_{x \to 0} \frac{x^2(5x+8)}{x^2(3x^2 - 16)} = -\frac{1}{2}$

8) $\lim_{x \to 0} \frac{\sin 2x}{x} \cdot \frac{2}{2}$

$\lim_{x \to 0} \frac{2 \sin 2x}{2x}$

$2 \cdot \lim_{x \to 0} \frac{\sin 2x}{2x} = 2$

9) $\lim_{x \to 0} \frac{\sin^2 x}{x}$

$\lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \sin x$

$1 \cdot 0$

0

10) $\lim_{x \to 2} \frac{x+1}{x^2 - 4}$ DNE

11) $\lim_{x \to 3} \frac{x^2 - 9}{x + 3} = 0$

12) $\lim_{x \to 0^-} \frac{x}{|x|} = -1$

13) a) $\lim_{x \to 3^-} f(x) = 3$

b) $\lim_{x \to 3^+} f(x) = -2$

c) $\lim_{x \to 3} f(x)$ DNE

d) $f(3) = 1$

14) a) $\lim_{s \to -2^-} p(s) = 3$

b) $\lim_{s \to -2^+} p(s) = 3$

c) $\lim_{s \to -2} p(s) = 3$

d) $p(-2) = 3$

15) a) $\lim_{x\to 4}(g(x)+3)$

$\lim_{x\to 4} g(x) + \lim_{x\to 4} 3$

$3 + 3$

6

b) $\lim_{x\to 4} \frac{g(x)}{f(x)-1}$

$\frac{\lim_{x\to 4} g(x)}{\lim_{x\to 4} f(x) - 1}$

$\boxed{-3}$

c) $\lim_{x\to 4} g^2(x)$

$\left[\lim_{x\to 4} g(x)\right]^2$

$(3)^2 = \boxed{9}$

16) $f(x) = \begin{cases} 3-x, & x<2 \\ \frac{x}{2}+1, & x>2 \end{cases}$

$\lim_{x\to 2^-}(3-x) = 1 \neq \lim_{x\to 2^+}\left(\frac{x}{2}+1\right) = 2$

$\lim_{x\to 2} f(x)$ DNE

17) $f(x) = \begin{cases} 3-x, & x<2 \\ 2, & x=2 \\ \frac{x}{2}, & x>2 \end{cases}$

$\lim_{x\to 2^-}(3-x) = 1 = \lim_{x\to 2^+}\left(\frac{x}{2}\right) = 1$

$\lim_{x\to 2} f(x) = 1$

18) $\lim_{x\to c} f(x)$ exists for $[0,1)\cup(1,2)\cup(2,\infty)$

19) $\lim_{x\to c} f(x)$ exists for $[-2\pi, 0)\cup(0, 2\pi]$

20) $\lim_{x\to 0}\left(x \sin\frac{1}{x}\right)$

$\lim_{x\to 0} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = 1$